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Calculation of the turbulent flow in plane diffusers

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Received 9 August 2005
Revised 23 January 2006
Accepted 23 January 2006

Abstract

Purpose – To provide an analysis of turbulent flow in plane diffusers for graduate and postgraduate students (researchers) which can help them to understand turbulent flows and turbulence modelling.

Design/methodology/approach – Steady, incompressible, turbulent flow in two-dimensional plane diffusers, where Reynolds averaged Navier-Stokes (RANS) equations were simplified using the theory of turbulent boundary layers in integral form adjusted for the internal flow. To close the RANS equations, the mixing length model proposed by Michel *et al.*, which was previously used for the calculation of turbulent flow in a straight channel with a uniform cross section, is applied for the calculation of the turbulent flow in plane diffusers. Also, in this paper, the velocity profile is approximated in every cross-section of the diffuser by a six-order polynomial, whose coefficients depend upon the three form parameters. Using this transformation, the system of governing equations was reduced to the three ordinary differential equations which were solved numerically.

Findings – A comparison between results obtained (velocity profiles) and experimental data obtained using HWA and LDA shows very good agreement. The method of integral equations of boundary layer is a relatively old method and tends to be forgotten since more advanced methods have been introduced. However, the results obtained using this method for the calculation of turbulent flow in a plane diffuser show a very good agreement with experimental data. Therefore, in engineering applications when simplicity and low-cpu times are required, the integral method can still be applied successfully.

Originality/value – This paper offers practical help to an individual starting his/her research in the computational fluid dynamics (turbulence modelling).

Keywords Turbulent flow, Fluid dynamics, Modelling

Paper type Research paper

Nomenclature

C_p	= pressure recovery coefficient	p	= pressure
C_f	= friction coefficient	$Re = uL/\nu$	= Reynolds number
HWA	= Hot Wire Anemometry	$u^*(x) = \sqrt{\tau_\omega(x)/\rho}$	= friction velocity
LDA	= Laser Doppler Anemometry	u^+	= dimensionless velocity
l	= mixing length	u_m	= mean velocity
		$\overline{u'_j u'_i}$	= Reynolds stresses
		$\dot{V}$	= flow rate

This paper presents the results of the work which has been previously done by Dr Mile R. Vujičić during his work at Faculty of Electro Technical Engineering, University of Serb Sarajevo, Serbs Republic.

International Journal of Numerical Methods for Heat & Fluid Flow
Vol. 17 No. 5, 2007
pp. 533-547

0961-5539
DOI 10.1108/09615530710752991

x	= co-ordinate along the diffuser axis
x_s	= dimensionless co-ordinate along the diffuser axis measured from the diffuser inlet to the point of the separation of the boundary layer
y, y^+	= co-ordinate normal to the diffuser axis with an origin at the diffuser axis and its dimensionless value
Greek symbols	
δ, δ^+	= distance from the diffuser axis to the wall of the diffuser and its dimensionless value
δ_0	= distance from the diffuser axis to the wall of the diffuser at the diffuser inlet
$\delta_1, \delta_2, \delta_3$	= displacement thickness, momentum thickness and energy thickness
$\eta, \eta^+ = \eta u^*/\nu$	= co-ordinate normal to the diffuser axis with an origin at the wall of the diffuser, and its dimensionless value
θ	= semi-angle of the diffuser
λ, q	= form parameters
ν	= kinematic viscosity
ν_t	= turbulent viscosity
ρ	= density
τ_w	= wall shear stress
Superscripts	
$'$	= derivative with respect to x
*	= symbol used for friction velocity
+	= dimensionless quantities
Subscripts	
0	= inlet cross section of diffuser
e	= axis of diffuser
m	= mean value
s	= point of separation of boundary layer
w	= diffuser wall

1. Introduction

A diffuser, as an element where the stream cross section changes from inlet to outlet has great importance as an adapter for a pipe-line, or in an ejector for changing velocity and pressure, or in a chimney, etc. The flow structure in the diffuser, whether laminar (Crnojević 1993) or turbulent has been the theme of many investigations in last 60 years. Up to the 1980s, turbulent shear flow was solved using the theory of a turbulent boundary layer with mixing length modelling (Johnston, 1998). Later, more advanced methods such as k-ε (Ganesan *et al.*, 1991) and large-eddy simulation (Gatski *et al.*, 1996; Ferziger and Perić 2002; Geurts, 2004; Schluter *et al.*, 2005) for computing turbulent flows have been introduced. Also, direct numerical simulation (DNS), which gives very detailed information about the flow, has been developed with fast parallel supercomputers. Unfortunately, DNS is too expensive (in terms of cpu time and memory required) to be used as a design tool and is limited to low-Reynolds numbers.

The analysis presented in this paper includes: calculations of the velocity profile from the diffuser inlet to the point of the separation of the boundary layer (the point of separation of the boundary layer is defined with a zero value of tangential stress on the wall); defining flow regimes (depending on the geometry and Reynolds number) which are defined in Kline's diagram (Fox and Kline, 1962); and calculations of friction and pressure recovery coefficients. Experimental data for pressure recovery coefficient was obtained by Ganesan *et al.* (1991), while experimental data for the friction coefficient was obtained by Johnston (1998).

The problem examined in this paper has been analyzed experimentally (using HWA or LDA) as well as numerically (finite difference, finite element or finite volume method). However, very rarely is this problem solved by an integral boundary method, as presented by Johnston (1998). It is well known that integral equations of the boundary layer, as analysed by Johnston (1998) and used for the computation of the turbulent flow, require an approximation of the velocity profile. Therefore, several different approaches for the approximation of the velocity profiles have been proposed. One of these approaches is a velocity deficit, which is based on the transverse coordinate and the boundary layer displacement thickness, and is described by an asymptotic seventh-order series (Singh and Azad, 1995) or by a sine function (Perry and Schofield, 1973). As mentioned earlier, an approximation of the velocity profile by the six-order polynomial based in the eddy turbulent viscosity is used in this paper. The final result of the simplifications applied on the equations presented below gives a system of non-linear simple differential equations, which is solved by a fourth order Runge-Kutta method.

2. Governing equations

Two-dimensional steady turbulent flow in a diffuser, shown in Figure 1, is described by Navier-Stokes and continuity equations:

$$u_j \partial_j u_i = -\frac{1}{\rho}\partial_i p + \nu \nabla^2 u_i \tag{1}$$

$$\partial_i u_i = 0 \tag{2}$$

Velocity decomposition into a sum of its mean and a fluctuation ($u = \bar{u} + u'$) applied on the equations (1) and (2) gives Reynolds averaged Navier-Stokes (RANS) equations and the equation of continuity for mean velocity:

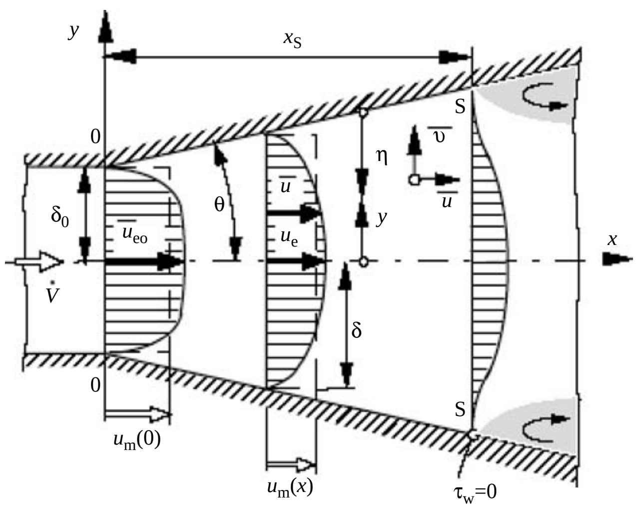


Figure 1.
Flow through a diffuser

$$\bar{u}_j \partial_j \bar{u}_i = -\frac{1}{\rho}\partial_i \bar{p} + \nu\nabla^2 \bar{u}_i - \partial_j \overline{u'_j u'_i} \tag{3}$$

$$\partial_i \bar{u}_i = 0 \tag{4}$$

Further, simplification of equation (1) is made using turbulent boundary layer theory presented in Cebeci and Cousteix (1999). This simplification includes the following assumptions:

- streamwise gradients are much smaller than cross-stream gradients ($\partial/\partial x \ll \partial/\partial y$); and
- $(1/\rho)(\partial\bar{p}/\partial y)$ is dominant in the momentum equation in y-direction.

Finally, equations (3) and (4) can be written in common notation:

$$\bar{u}\frac{\partial \bar{u}}{\partial x} + \bar{v}\frac{\partial \bar{u}}{\partial y} = -\frac{1}{\rho}\frac{\partial \bar{p}}{\partial x} + \frac{\partial}{\partial y}\left[(\nu + \nu_t)\frac{\partial \bar{u}}{\partial y}\right] \tag{5}$$

$$\frac{1}{\rho}\frac{\partial \bar{p}}{\partial y} = 0 \tag{6}$$

$$\frac{\partial \bar{u}}{\partial x} + \frac{\partial \bar{v}}{\partial y} = 0 \tag{7}$$

where ν_t presents turbulent viscosity (introduced by Boussinesq).

Equations (5), (6) and (7) are applied from the diffuser inlet 0-0, to the cross section s-s where the boundary layer starts to separate ($\tau_w = -\rho\nu(\partial\bar{u}/\partial y)_{y=0} = 0$). These equations are solved by satisfying the boundary conditions:

$$y = 0, \bar{u}(x,0) = \bar{u}_e(x), \quad \partial\bar{u}/\partial y = 0; \tag{8}$$

$$y = \delta, \bar{u}(x,\delta) = 0, \quad \bar{v}(x,y) = 0 \tag{9}$$

$$\dot{V} = 2\int_0^\delta \bar{u}\,\mathrm{d}y = \text{const.} \tag{10}$$

$$0 = \bar{u}_e \bar{u'}_e + \frac{\partial}{\partial y}\left[(\nu + \nu_t)\frac{\partial \bar{u}}{\partial y}\right]_\delta - \nu\left(\frac{\partial^2 \bar{u}}{\partial y^2}\right)_e \tag{11}$$

Boundary conditions (8) and (9) define velocity on the diffuser axis and for no-slip velocity, respectively. The boundary condition (11) was obtained by satisfying the momentum equation (5) for the diffuser axis (subscript e) and for the channel wall (subscript δ), whereby the condition for turbulent stress on the axis $(\rho\nu_t\partial\bar{u}/\partial y)_e = 0$ has been employed (Figure 2).

If one applies the classical procedure for transition on integral equations, then partial differential equations (5), (6) and (7) will take the form:

$$\frac{\mathrm{d}\delta_2}{\mathrm{d}x} + (2\delta_2 + \delta_1)\frac{\bar{u}'_e}{\bar{u}_e} = \frac{\tau_w}{\rho\bar{u}_e^2} + \frac{\nu\delta}{\bar{u}_e^2}\left(\frac{\partial^2 \bar{u}}{\partial y^2}\right)_e \tag{12}$$

$$\frac{d\delta_3}{dx} + 3\delta_3 \frac{\bar{u}'_e}{\bar{u}_e} = 2\int_0^\delta \frac{\bar{u}}{\bar{u}_e}\left[\frac{\nu}{\bar{u}_e^2}\left(\frac{\partial^2 \bar{u}}{\partial y^2}\right)_e - \frac{\partial}{\partial y}\left(\frac{\tau}{\rho \bar{u}_e^2}\right)\right]dy \quad (13)$$

where equation (12) represents the momentum equation and equation (13) represents the mechanical energy equation, where the total shear stress τ is identified as:

$$\tau = \rho(\nu + \nu_t)\frac{\partial \bar{u}}{\partial y} \quad (14)$$

where the variable $\bar{u}'_e$ is the derivative $\bar{u}'_e = d\bar{u}_e/dx$, and δ_1, δ_2 and δ_3 are displacement thickness, momentum thickness and energy thickness, respectively, defined as:

$$\delta_1 = \int_0^\delta \left(1 - \frac{\bar{u}}{\bar{u}_e}\right)$$

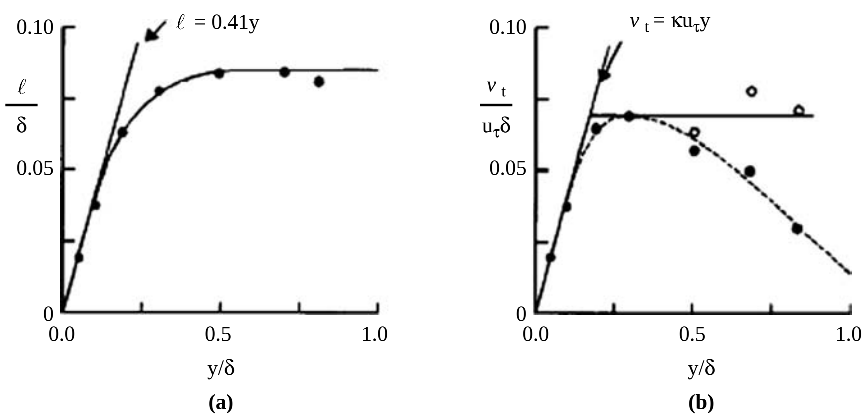
$$\delta_2 = \int_0^\delta \frac{\bar{u}}{\bar{u}_e}\left(1 - \frac{\bar{u}}{\bar{u}_e}\right)$$

$$\delta_3 = \int_0^\delta \frac{\bar{u}}{\bar{u}_e}\left[1 - \left(\frac{\bar{u}}{\bar{u}_e}\right)^2\right]dy \quad (15)$$

With the aim of solving equations (12) and (13), the velocity profile is approximated by the six-order polynomial:

$$u^+(x,y) = a(x) + b(x)y^{+2} + c(x)y^{+4} + d(x)y^{+6} \quad (16)$$

To ensure a symmetrical velocity profile, only even power ratios of the polynomial were used. An analysis of turbulent flow shows that it is useful if one introduces a coordinate measured positive from the wall $\eta = \delta\text{-}y$, friction velocity $u^*(x) = \sqrt{\tau_w(x)/\rho}$ (where τ_w is shear stress on the wall) and dimensionless variables:



Note: The symbols denote the experimental data from Klebanoff, 1954

Figure 2. Dimensionless (a) mixing-length and (b) eddy viscosity distributions across a turbulent boundary layer in a zero pressure gradient $u_e\theta/\nu \cong 8{,}000$

$$u^+ = \frac{\bar{u}}{u^*}, \quad y^+ = \frac{yu^*}{\nu} = \delta^+ - \eta^+$$
$$\eta^+ = \frac{\eta u^*}{\nu}, \quad \delta^+ = \frac{\delta u^*}{\nu} \tag{17}$$

If one uses the velocity profile (16) and satisfies boundary conditions (8)-(11), polynomial coefficients will be determined as:

$$a(x) = u_e^+ \tag{18}$$

$$b(x) = \frac{-8q + 5.25\,\mathrm{Re} - 0.0333\lambda\delta^{+3}}{\delta^{+3}} \tag{19}$$

$$c(x) = \frac{70q - 52.5\,\mathrm{Re} + 0.667\lambda\delta^{+3}}{\delta^{+5}} \tag{20}$$

$$d(x) = \frac{-4.67q + 3.5\,\mathrm{Re} - 0.0778\lambda\delta^{+3}}{\delta^{+7}} \tag{21}$$

where $\mathrm{Re} = 2\delta\bar{u}_e/\nu$ is the Reynolds number, and the form parameters are:

$$\lambda(x) = \frac{\bar{u}_e\bar{u}_e'\nu}{u^{*3}} = \frac{\bar{u}_e'\delta^2}{\nu}\frac{u_e^+}{\delta^{+2}}, \quad q(x) = \delta^+\bar{u}_e^+ \tag{22}$$

Using formula (22), the relationship between the form parameters is obtained:

$$q' = \frac{1}{\delta q}(\lambda\delta^{+3} + q^2\delta') \tag{23}$$

The last term in equation (3) is a second order tensor or a matrix in any particular coordinate system, and represents the average of the products of the fluctuation velocity components (Durbin and Pettersson Reif, 2001):

$$\overline{\mathbf{u}_j'\mathbf{u}_i'} = \begin{bmatrix} \overline{u_1'u_1'} & u_1'u_2' & u_1'u_3' \\ u_2'u_1' & u_2'u_2' & u_2'u_3' \\ u_3'u_1' & u_3'u_2' & u_3'u_3' \end{bmatrix} \tag{24}$$

Matrix (24) is called the *Reynolds stress tensor*.

The Navier-Stokes equations have a quadric nonlinearity, and they are unclosed. To close the Navier-Stokes equations, the mixing length model where $\nu_t = l^2 \mathrm{d}\bar{u}/\mathrm{d}\eta$, introduced by Prandtl, is applied. The mixing length $l(\eta)$ defined by Michel *et al.* (Cebeci and Cousteix, 1999) is used:

$$\frac{l(\eta)}{\delta} = 0.085\tan h\left(\frac{\kappa}{0.085}\frac{\eta}{\delta}\right) \tag{25}$$

where $\kappa = 0.4$. For simplicity, the expression (25) is approximated by the fourth-order polynomial:

$$\frac{l}{\delta} = 0.472\frac{\eta}{\delta} - 0.98\left(\frac{\eta}{\delta}\right)^2 + 0.894\left(\frac{\eta}{\delta}\right)^3 - 0.301\left(\frac{\eta}{\delta}\right)^4 \tag{26}$$

By linearisation, the expression (26) is not reduced to the well-known Prandtl's expression for the mixing length: $l = \kappa y$, but in spite of that, it approximates the function (25) very well in the whole cross section.

Mathematical manipulation, which included differentiations and integrations (this mathematical work has been carried out in *Mathematica 5*), and is dictated by equations (12) and (13) and the formula (15), produces a system of three simple differential equations:

$$\frac{\mathrm{d}\lambda}{\mathrm{d}x} = f(x, \lambda, q, \delta^+), \quad \frac{\mathrm{d}q}{\mathrm{d}x} = g(x, \lambda, q, \delta^+), \quad \frac{\mathrm{d}g^+}{\mathrm{d}x} = h(x, \lambda, q, \delta^+) \tag{27}$$

A prediction that the turbulent velocity profile is fully developed at the diffuser inlet, where $\bar{u}_e'(0) = 0$, defines that parametrical form λ is found to be zero. However, experimental results, as in Vujičić (2001) show that the fluid stream adapts before entering the diffuser inlet and follows the geometry of the diffuser, the outcome of which is:

$$\lambda(0) \neq 0 \tag{28}$$

but which is very close zero. The form parameter q is defined as $q = (\bar{u}_e/u_\mathrm{m})\mathrm{Re}/2$. In this expression, the Reynolds number is defined as $\mathrm{Re} = 2\delta u_\mathrm{m}/\nu$, while u_m denotes mean velocity in the cross section of the diffuser. Using a well-known relation between velocities, $u^*/u_\mathrm{m} = \sqrt{C_\mathrm{f}/2}$ where C_f is friction coefficient, the initial condition of the form parameter q is obtained:

$$q(0) = \frac{\mathrm{Re}}{2}\left(1 + 3.75\sqrt{\frac{C_\mathrm{f}(0)}{2}}\right) \tag{29}$$

The third initial condition is determined using definition (17) where the variable δ^+ is defined as:

$$\delta^+(0) = \frac{\mathrm{Re}}{2}\sqrt{\frac{C_\mathrm{f}(0)}{2}} \tag{30}$$

The initial value of friction coefficient is determined using the Blasisus formula:

$$C_\mathrm{f} = \frac{1}{4}\left(\frac{0.3164}{\sqrt[4]{\mathrm{Re}}}\right) \tag{31}$$

The system of differential equations (27) is solved by the Runge-Kutta method of fourth order. As mentioned earlier, numerical calculations are stopped in the downstream cross section where the flow starts to separate from the wall of the diffuser. In this regard, an appropriate computer program has been written in Fortran 77.

3. Pressure and friction coefficients

In order to have a detailed knowledge of the flow structure in a diffuser, it is necessary to know both velocity and pressure fields which are defined by the global parameters: pressure recovery coefficient (C_p), friction factor (C_f) and the local loss of energy.

These parameters are the subject of interest in engineering applications. Pressure recovery coefficient is defined as:

$$C_{\mathrm{p}}(x) = \frac{p(x) - p(0)}{\rho u_{\mathrm{m}}^2(0)/2} = 1 - \left(\frac{u_{\mathrm{m}}(x)}{u_{\mathrm{m}}(0)}\right) \tag{32}$$

where subscript m represents the mean value in the cross section. The friction coefficient is determined using the following expression:

$$C_{\mathrm{f}}(x) = 2\tau_{\mathrm{w}}(x)/\rho u_{\mathrm{m}}^2(x) \tag{33}$$

where the mean value of velocity in the cross section at the distance x is determined as:

$$u_{\mathrm{m}}(x) = \delta^{-1}(x)\int_0^{\delta} \bar{u}(x, y)\mathrm{d}y \tag{34}$$

4. Numerical results and discussion

In the order to have concrete numerical results, one has to define the geometry of the diffuser and the value of the Reynolds number at the diffuser inlet. In this paper, the geometry of the diffuser is defined as a straight walled slope of half-angle θ, and the cross section change is defined by the linear function:

$$\delta(x) = \delta_0 + x \cdot \tan\theta \tag{35}$$

Figures 3 and 4 show the results of the development of the velocity profile, defined in relation to the maximal velocity of the inlet cross section, for the value of Reynolds number Re = 50,000 and half-angles of the diffuser $\theta = 15°$ (Figure 3) and $\theta = 30°$ (Figure 4). From these diagrams, the development of the inlet velocity profile to the velocity profile at the separation point of the boundary layer can be clearly seen.

Figures 3 and 4 show that the velocity profile in the inlet cross section at the axis of the diffuser deviates from the fully developed one for approximately 5 per cent, which is a consequence of the inlet boundary condition (28).

After the separation point in the diffuser, a different type of turbulent flow begins, which cannot be described using the current model. If one compares velocity profiles for half-angles $\theta = 15°$ with $x_{\mathrm{s}}/\delta_0 = 1.445$ and $\theta = 30°$ with $x_{\mathrm{s}}/\delta_0 = 0.66$ using a constant Reynolds number, then one can see that the position of the separation point moves nearer to the diffuser inlet as the angle of the diffuser increases.

Figure 5 shows the details of the pressure recovery coefficient distribution along the length of the diffuser determined by expression (32). From Figure 5, one can see that the value of the pressure increases along the length of the diffuser and its change is identified by the pressure recovery coefficient, with more intensity for a diffuser with a greater angle.

Figure 6 shows a comparison between the results obtained for pressure recovery coefficient, for a half-angle of the diffuser $\theta = 4°$ with the results from Ganesan *et al.* (1991), in which the problem of turbulent flow in a plane diffuser is solved using both k-ε and Prandtl's mixing length model, with uniform distribution in some parts of the cross section. From this figure, one can see that both models defined by Prandtl's mixing length gave similar results, although a difference does exist which is probably

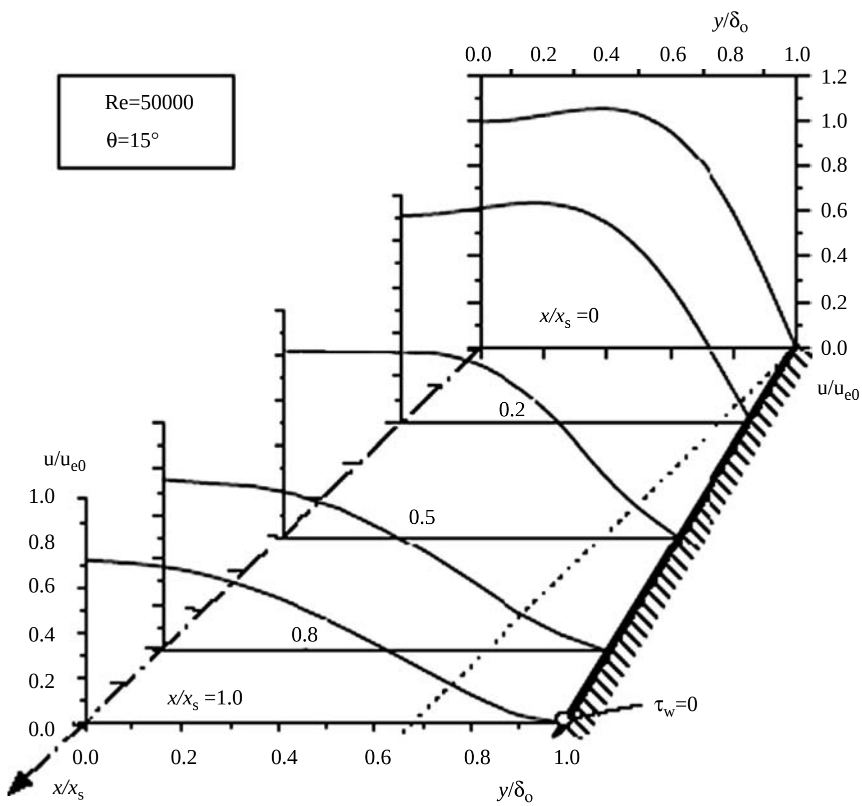


Figure 3.
Velocity profiles in a diffuser at Re $=$ 50,000 and for $\theta = 15°$; $x_s/\delta_0 = 1.445$

a consequence of the uniform distribution of mixing length mentioned in the original reference. Here, as in Ganesan *et al.* (1991), there exists a difference in values of pressure recovery coefficient which are obtained in k-ε model and Prandtl's mixing length model.

The friction coefficient can be determined using the velocity field obtained with expressions (34) and (14). Also, it is known that the friction coefficient is highly dependent on the angle of a diffuser. Therefore, a dependence of friction coefficient on an angle θ for the flow determined by Re $=$ 10,000, is shown in Figure 7. A typical drop of the friction coefficient from the initial value C_f (0) to the value $C_f(x_s) = 0$ in the separation cross section is clearly noted, whereby it is obvious that the drop is more intense in the diffusers with larger angles.

A comparison of results obtained in non dimensional form for a plane diffuser at Re $=$ 6,000 and $\theta = 5°$ with the corresponding experimental results presented in Johnston (1998), is shown in Figure 8. As can be seen, Figure 8 shows very good agreement between the numerical and experimental results.

The comparison of velocity profiles in the separation cross section between prediction and experimental data is shown in Figures 9-11.

A comparison between the results obtained using integral equations presented in this paper and experimental data presented in the work of Stieglmeier *et al.* (1989) for

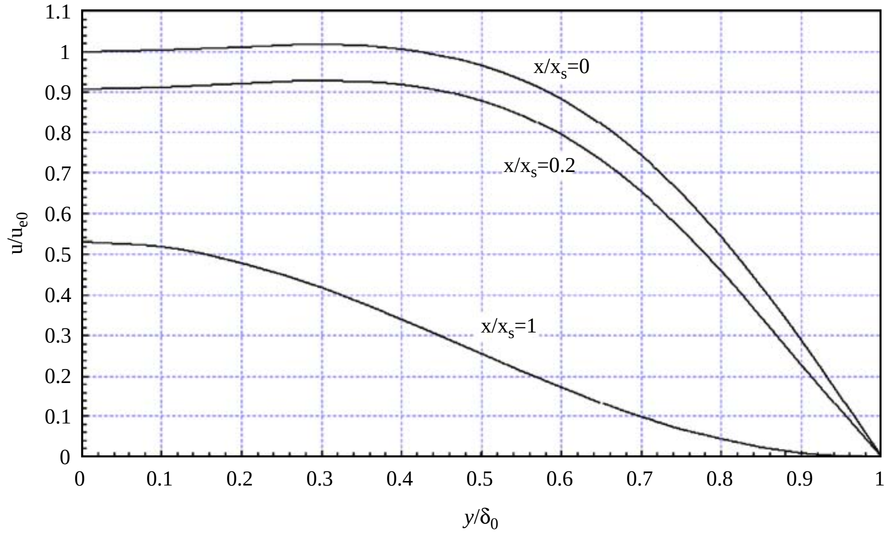


Figure 4.
Velocity profiles in a diffuser at Re $=$ 50,000 with semi-angle of $\theta = 30°$

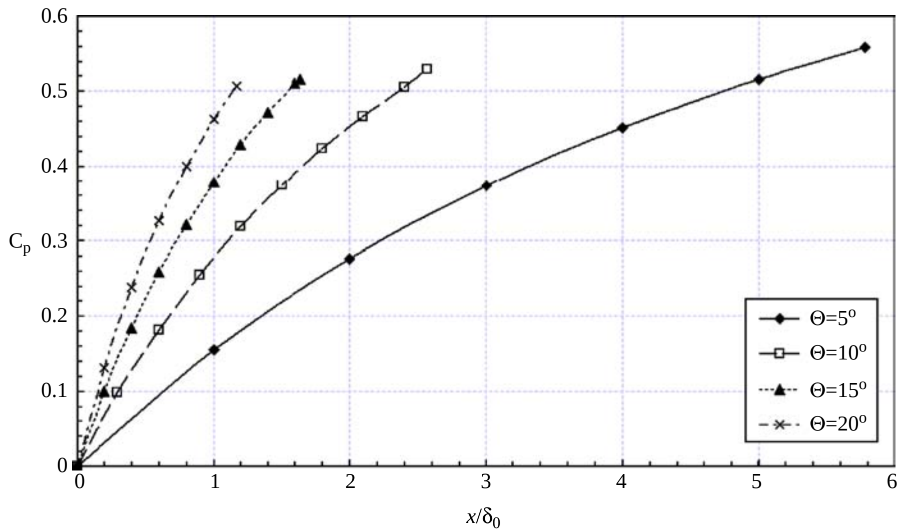


Figure 5.
Pressure recovery coefficient in a diffuser at Re $=$ 25,000 and for different values of θ

the diffuser with a geometry defined by an angle of 28° and flow determined by Re $=$ 15,600 is shown in Figure 9. Experimental results presented in the work of Stieglmeier *et al.* (1989) are obtained using LDA. From Figure 9, an excellent agreement between numerical results and experimental data can be seen. A further comparison between numerical results and experimental data presented in the work of Stieglmeier *et al.* (1989) includes a diffuser geometry defined by an angle of 36° and same Reynolds number as in the previous case (Figure 10). An excellent agreement as well as in the previous case can be shown in Figure 10.

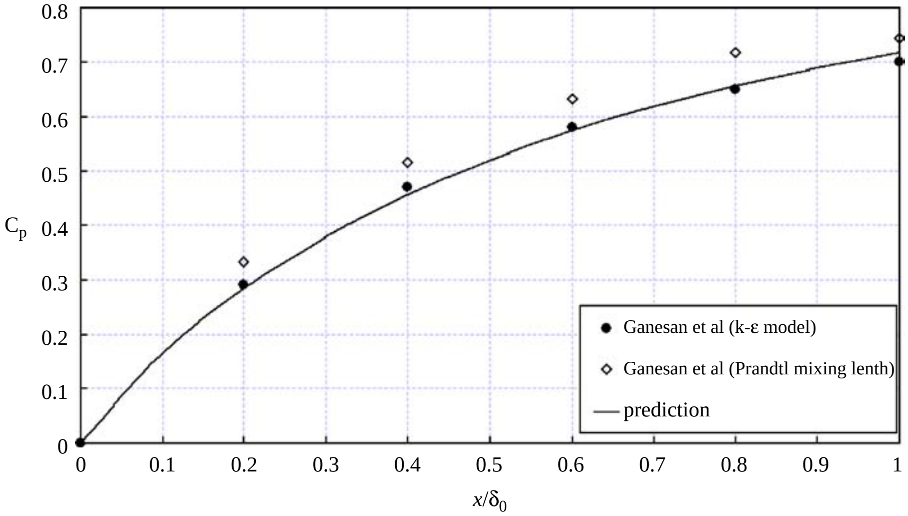


Figure 6.
Comparison of results with reference Ganesan *et al.* (1991) for $\theta = 4°$

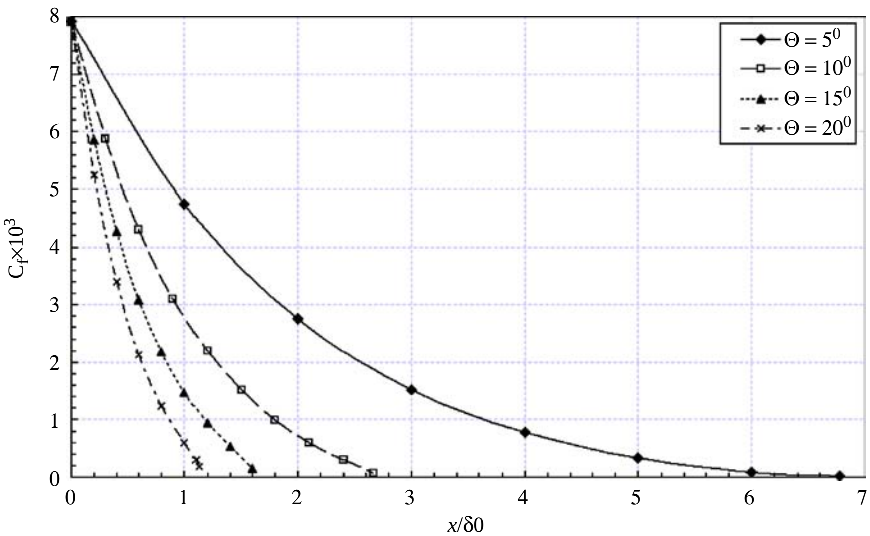


Figure 7.
Friction coefficient in a diffuser at Re = 10,000 and for different values of θ

The third comparison shown in Figure 11 includes experimental data from Singh and Azad (1995). Singh and Azad (1995) used a HWA to measure velocity profiles in a diffuser described by an angle of 8°. Turbulent flow in the diffuser was defined by Re = 69,000. From Figure 11, very good agreement between numerical results and experimental data can be seen.

4.1 Point of separation of the boundary layer

Determination of the point of separation of the boundary layer is very important for optimisation of diffuser geometry. It has been noted that a point of separation of the

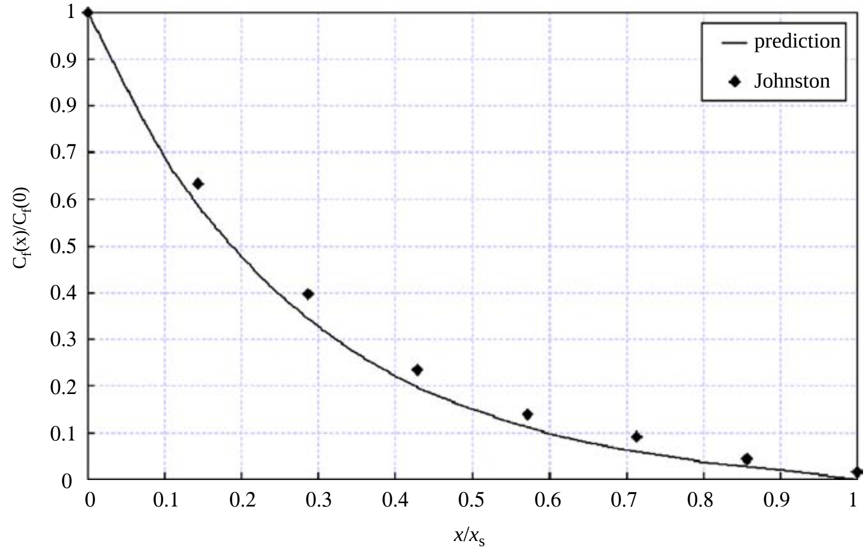


Figure 8.
Comparison between friction factors for a diffuser of $\theta = 5°$ at Re = 6,000

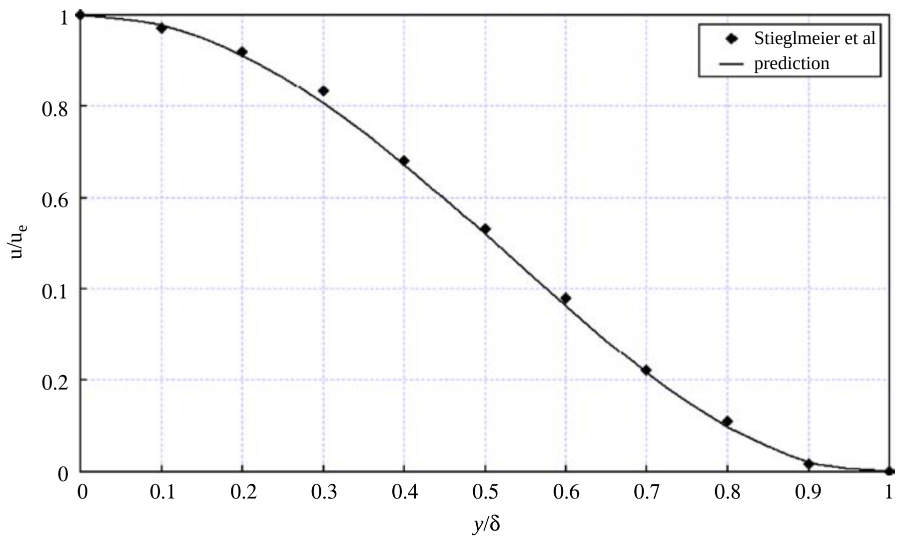


Figure 9.
Velocity profiles in the cross section with separation flow for Re = 15,600 and for $\theta = 14°$

boundary layer is governed by variations of Reynolds number and different angles of a diffuser. Therefore, the whole spectrum of different separation regimes of the flow is obtained, shown in Figure 12 and compared with Kline's diagram, where $n = x_s/\delta_0$ is the characteristic parameter.

From Figure 12, it can be seen the most of the calculated flow regimes belong to a region of transitional flow in Kline's diagram. Also, from Figure 12, a logical behaviour of the separation point is noted: for a fixed Reynolds number, the separation is

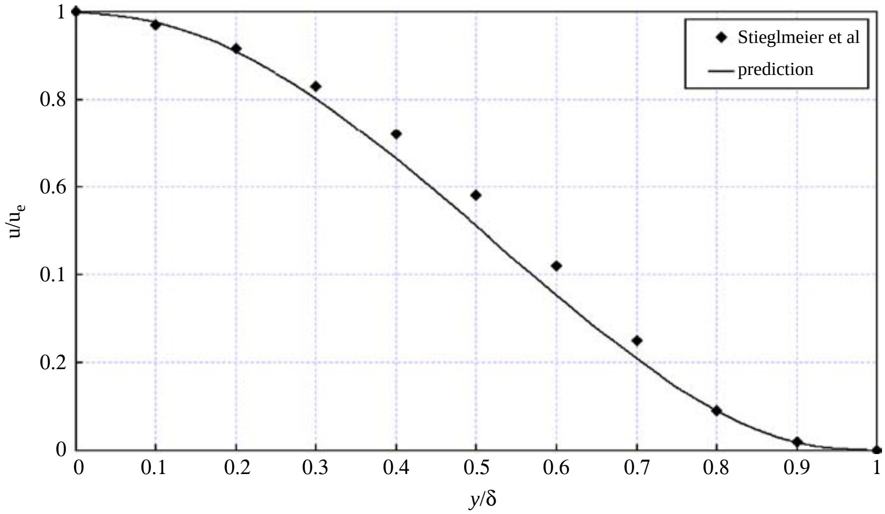


Figure 10.
Velocity profiles in the cross section with separation flow for Re = 15,600 and for $\theta = 18°$

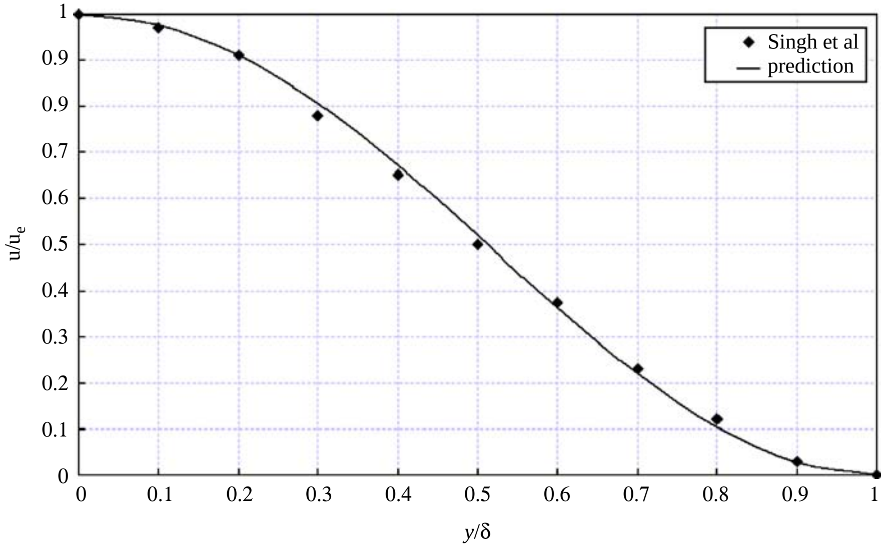


Figure 11.
Velocity profiles in the cross section with separation flow for Re = 69,000 and for $\theta = 4°$

enhanced with the increase of the angle, while for a fixed diffuser angle the separation is enhanced with the increase of the Reynolds number. According to Johnston (1998), the optimal region is defined by $\delta_s/\delta_0 = 2$ up to $\delta_s/\delta_0 = 4$, and n between 5 and 15. It is seen that optimal parameters can be achieved for relatively small values of the Reynolds number and the diffuser angle. As mentioned earlier, the results presented can be used for the choice of optimal dimensions of plane diffusers in the case in which no flow separation occurs.

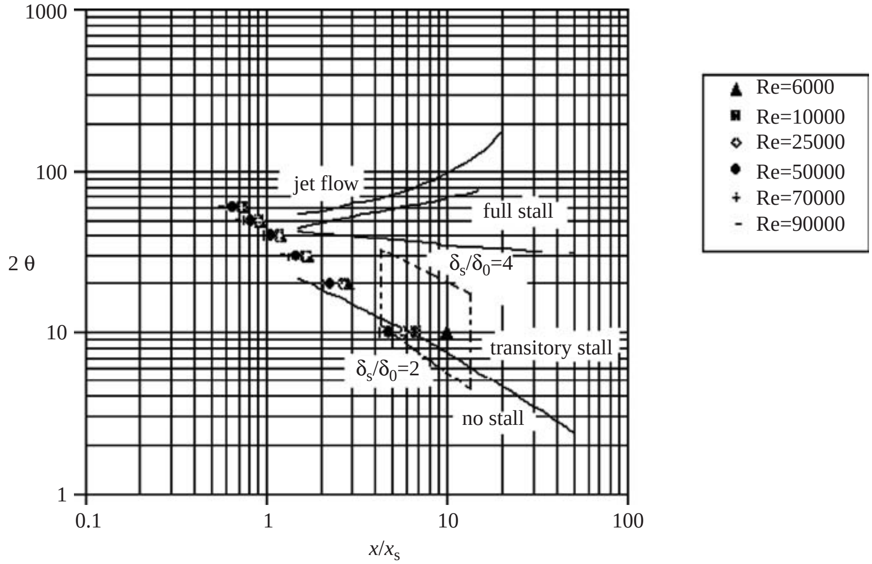


Figure 12.
Separation distance between inlet and separation point for different angles and Reynolds numbers

5. Conclusions

The method of integral equations of boundary layer theory, suitably adjusted for the calculation of turbulent flow in plane diffusers is presented in this paper. The results obtained contain the development of velocity profiles and changes of pressure recovery coefficient and friction coefficient from the inlet cross section to the separation cross section of the diffuser. It is known that these quantities are functions of the Reynolds number and the diffuser angle, and their changes are more intense for greater angles of diffusers. From the results presented in this paper, it can be noted that for fixed values of Reynolds number, the position of the separation cross section is postponed for smaller diffuser angles. Comparisons between results predicted in this paper and experimental data obtained by Stieglmeier *et al.* (1989) and Singh and Azad (1995) show a very good agreement, and that deviations between them are relatively small.

The method of integral equations of boundary layer is a relatively old method and tends to be forgotten since more advanced methods have been introduced. However, the results obtained using this method for the calculation of turbulent flow in a plane diffuser show a very good agreement with experimental data. Therefore, in engineering applications when simplicity and low-cpu times are required the integral method can still be applied successfully. Also, it has been noted that for relatively large angles of diffuser, the basic hypothesis of boundary layer theory may become invalid. This is the reason why the proposed method for calculation of turbulent flow in plane diffusers is not recommended for relatively large angles of diffuser.

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